Theoretical and Methodological Learning Issues as Products of Innovative Design and Implementation

תאוריה ומתודולוגיה כתוצרים של מחקר המלווה פיתוח והפעלה של פרויקט חדשני - רינה הרשקוביץ

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The Practical Situation…

- The power of computerized tools for learning mathematics is a welcomed idea (around 1993).

- → Budget → The best team → The CompuMath project for Junior High-School Curricula.

- Designed activities as milestones for the Curricula’s themes

- As practitioners, the project team members were overwhelmed and surprised by what they observed in trial classrooms. The need to meaningfully understand, describe and explain the observed learning practices, raised the need for an operational definitions.

The Theoretical Situation…

- Abstraction has been the focus of extensive interest in several domains, including Mathematics Education. Many mathematics-educators and researchers have taken a predominantly theoretical stance and have described **abstraction as some type of de-contextualization**.

- ***Abstraction in Context*** **(AiC)** proposes a quite different approach to abstraction, which emerged from:

- The **Learning practices** which were observed.

- **Interweaving of theories** (e.g. Davydov, 1970-1990; Freudenthal and colleagues, 1995).

The AiC Theoretical Framework

AiC

(Dreyfus, Hershkowitz & Schwarz, 2001;Hershkowitz, Schwarz & Dreyfus, 2001).

- **Abstracting in context (AiC) --** a human activity of **"vertical mathematization"**, which represents the process of constructing a new construct in mathematics by learner/s within the mathematics itself and by mathematical means, including **insight** and **creativity.**

- **Abstraction is seen as reorganization of previous mathematical constructs** and interweaving them, into one process of mathematical thinking, with the purpose of constructing a new mathematical construct.

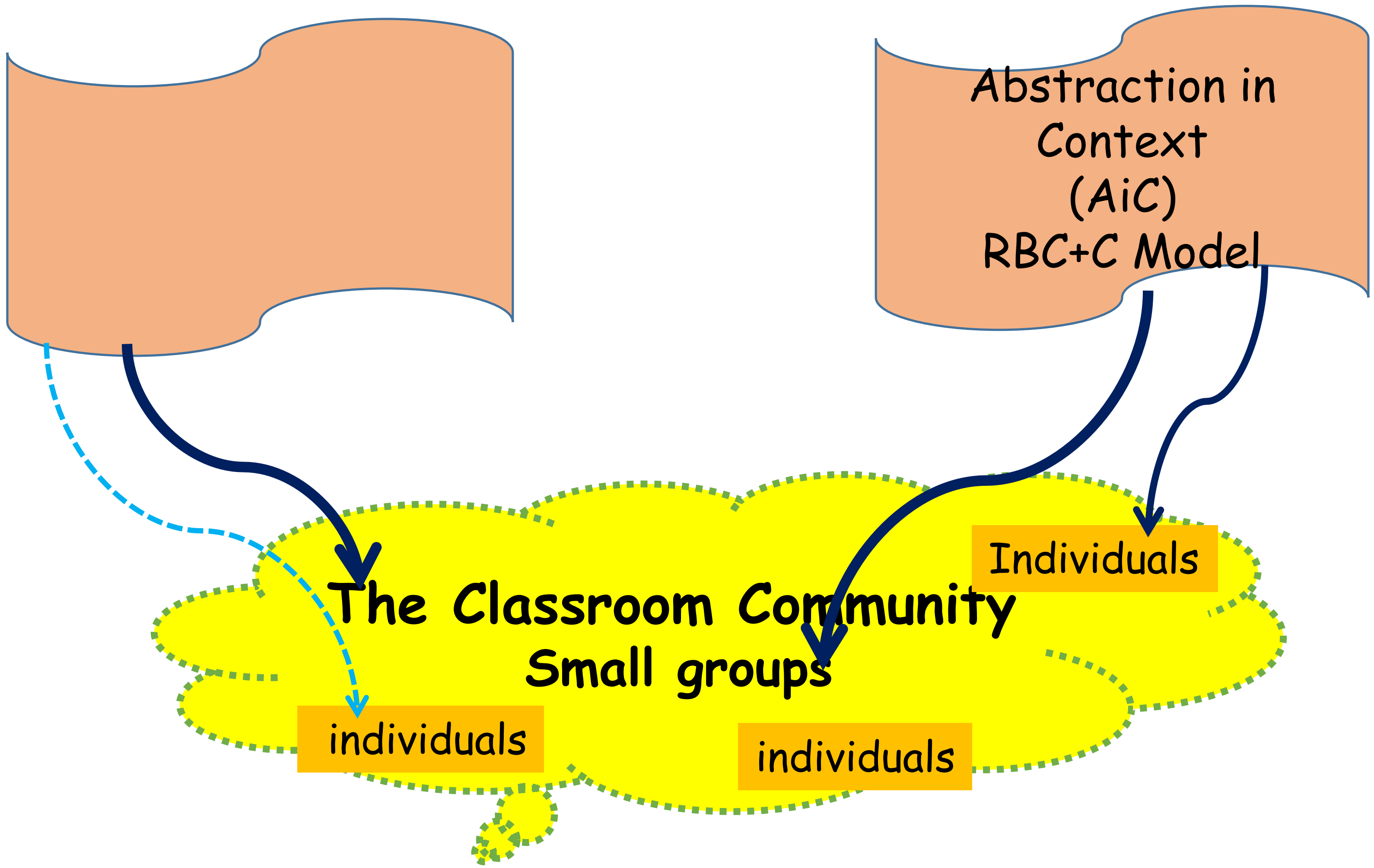
The nested RBC+C model

AiC & RBC+C

The model suggests **Constructing** as the central epistemic action of mathematical abstraction. **Constructing** of new knowledge is largely based on ***vertical re-organizing of existing knowledge elements*** in order to create **a new knowledge *construct***.

Recognizing - **The learner recognizes that a specific construct is relevant to the problem he or she is dealing with.**

Building-with - is an action **comprising the combination of recognized constructs in order to achieve a localized goal**, such as the actualization of a strategy or a justification or the solution of a problem.


Abstraction in
Context
(AiC)
RBC+C Model
The Classroom Community
Small groups
Individuals
individuals
individuals

Coordination

DCA

AiC & RBC+C

At PME 33 (2009) we had started to cooperate with the Documenting Collective Activity (DCA) approach.

"We" represents and coordinate now two approaches/methodologies that were working in parallel but separately:

- ✓ The Abstraction in Context (AiC) approach with the RBC+C model. Used for the analysis of processes of constructing knowledge by individuals and by small groups.
- ✓ The Documenting Collective Activity (DCA) approach with its methodology. Used for establishing normative ways of reasoning in the classroom's community.

Investigates the Normative Ways of Reasoning that are Developed in the Classroom Community

DCA

(Stephan & Rasmussen, 2002; Rasmussen & Stephan, 2008).

Such normative ways of reasoning emerge **as learners** are going through collective activities; **solve problems, explain their thinking, represent their ideas**, etc…

Collective activity is a sociological construct that addresses the constitution of ideas through patterns of interaction.

A mathematical idea or a way of reasoning becomes normative when there is **empirical evidence that this idea functions in the classroom as if it were shared.**

This emphasizes on empirical approach which uses Toulmin's model of argumentation (1958).

DCA

The main Research work with RBC+C & DCA

AiC & RBC+C

Constructing knowledge of:

Individuals in laboratory - 2001

Dyads in laboratory and in a working classroom2001

Small Groups in a working classroom – 2007

The whole class (With DCA) in a working classroom– 2013, 2014

Special characteristics:

❖The **micro** perspective

❖ The **longitudinal** perspective

The Focus of the Research Work last years:

Knowledge Shifts in Inquiry Classroom.

This means to better understand the mechanisms of knowledge shifts in the different settings of mathematics inquiry classroom, as it is expressed in **the roles of the students as individuals, as groups and as a community** in making these shifts.

And the role of the teacher in promoting and creating opportunities for such shifts.

Knowledge agents, followers, shifts of knowledge in the inquiry mathematics classroom

- **A *knowledge agent*** is a member in the classroom community who initiates a new idea, which subsequently is appropriated by one or several other members of the classroom community – ***The followers.***

- A student can **follow** an idea by repeating it, elaborating it, or objecting to it. The follower's action may occur immediately after the one of the knowledge agent. It may also take place in a later discussion, and/or in a different social setting in the class, or even in a later lesson.

- **A first shift of knowledge** takes place between the knowledge agent and her or his followers. However as other students take part in the discussion, more shifts may occur.

The goal

Investigating knowledge shifts in the mathematics classroom: in the different settings in the classroom and the role of students as well as the teacher in making these shifts happen.

In the whole class community: A few "avenues" of investigations:

Constructing a new knowledge in various topics (E.g. Introduction to Probability in Junior high-School, Self-Similarity in M.A. students),   and learning environments (E.g. Creative, With computerized tools).

Example:The Dreidel Problem Situation

Hanukkah dreidel (a four-sided top with one of the letters N, G, H, and P on each side) was spun 100 times. Mark approximately, on the chance bar, the chances of the following events:

A. The dreidel will fall 100 times on N.
B. The dreidel will never fall on N.
C. The dreidel will fall on N between 80–90 times.
D. The dreidel will fall on N between 20–30 times

The lesson opening

The teacher (T) opens the lesson reminding the students of the meaning of a ***chance bar*** and ***its boundaries***: **0** for the probability of an impossible event, **1** for the probability of a certain event.
T asks Itamar to mark the probability of event A on the chance-bar and he marks it close to ¼.

A. The dreidel will fall 100 times on N.

From Episode 1: The probability of Event A…

…

63 Itamar**:** That's correct.

64 T**:** All the 100 times, you marked that it is approximately a quarter of the cases.That's right, but, it's true that it is almost impossible, but relatively to the letter N there are four letters.

At 65 Itamar, expresses two contradictory claims.

From Episode 1: The probability of Event A…

68 T: What do you have to respond to that? (Repeats Itamar's argument that there are 4 letters but N is just one letter). Guy, how would you answer him?

69 Guy: There is, like, each time that you spin there is, like, 4 letters it can fall on, so each time it divides again by 4, the chance (Teacher: yes) and the chance decreases, it decreases each time that you spin that it will fall again on the same letter.

From Episode 1: The probability of Event A…

70 T: So you are reinforcing him?! You are saying that the mark is correct!?

71 Guy: No, you have to lower it.

72 T: Because…?

73 Guy: Because each time the probability is much smaller. when you spin twice and it falls on the same letter – the probability decreases.

Guy's justification raises **a mathematically correct new idea**, which resolves the conflict expressed by Itamar.
It explains clearly the meaning of the probability of event A as ***repeating event*** (69, 71 & 73).

We do not (yet) have an evidence for Guy being a knowledge agent, because we do not (yet) have followers to his idea.

From Episode 2: The probability of Event B

B. The dreidel will never fall on N ...

The goals: 1) to identify that event **B is a repeating event as well,** so its probability is decreasing as well.

2) to see if students will observe that in event A, the decrease is stronger than in event B, because in one spin **the chance that it will fall on N is 1/4, whereas the chances that it will not fall on N is 3/4.**

96 T: What do you think? Ayelet, what do you suggest?

97 Ayelet: B is a bit higher than A because it means that one of the other three letters will come out every time.

98 T: What do you think of Ayelet's explanation?

99 Students: Support her… [Many students raise their hands]...

100 Teacher: Support her, yes!

Interpretation -
B. The dreidel will never fall on N

- Ayelet's idea is based on the **Decreasing idea** expressed by Guy. Therefore, Ayelet's idea makes Guy **a knowledge agent** and makes Ayelet his **follower**. We have here an example for a **shift of knowledge** from a **knowledge agent** (Guy), through his **follower** (Ayelet), an input into the whole class discussion.
- In addition, Ayelet idea has a **new element** as well: **the chance that it will not fall on N is 3/4. This makes Ayelet a potential knowledge agent.** The support of many other students **makes her a knowledge agent.**
- Ayelet's idea is an evidence for a **new, mathematically anchored construct.**

Summary

- We showed that **shifts of knowledge** might be quite complex. Often they serve as **learning "milestones"** in the whole class discussion.

- In the example presented here in which a student (Guy) **expressed** his innovative mathematical idea which solved the given problem (A), this idea **was followed** by Ayelet only after a while. Ayelet elaborated this idea by her new idea, and the support of many other students completed this to two- steps knowledge shift.

- The mathematical idea **was downloaded to a working group** of three students at the end of the lesson.

Concluding Remarks

- **Both, Abstraction in Context (AiC), and Documenting Collective Activity (DCA) emphasize (1) the process of creation a new (to the student) piece of knowledge** and **(2) working within mathematics and by argumentation based on mathematical means.**
- **This enables us the coordination of the two Theoretical/Methodological frames into unique tool for investigating shifts of "just created new knowledge" in the mathematics inquiry classroom.**

- **Fortunately we had the curiosity and the initiative to perform this kind pieces of research with which we observed deeply the ways and mechanisms of constructing new knowledge in various mathematical themes by individuals students, groups and the collective.**

- **Others are interested (The encyclopedia for Mathematics Education).**

Tracing students’ knowledge construction and tracing shifts of the constructed knowledge in a classroom setting are challenges that still need to be achieved (Saxe et al.2009).

The End