The Limits of Physical Measurement

(Percival Lecture—1957)

By S. Devons

ABSTRACT

How small an object can be seen or measured? What limits are there to the smallest, or largest, length or time or other physical quantity, which has physical significance? Questions such as these have stayed in the forefront of physical science throughout its development. The physicists' attitude towards these problems has developed alongside the development of physics itself. In particular the extension of methods of measurement, as illustrated by the determination of size and shape of small objects, has been accompanied by extension and change of the concepts themselves. At the present time several factors can be recognised, and the most recently explored involved a range of magnitudes of 10^{42}: 1. Future extension of this range is likely to be accompanied by further development of our basic physical ideas, rather than by technical developments alone.

When I was honoured with the invitation to give the Percival Lecture, I had little hesitation in choosing a subject. To me as, I am sure, to many others, the name " Manchester Literary and Philosophical Society " is immediately linked with Dalton, and I have chosen a topic closely related to Dalton's contribution to science. Dalton dealt with the limiting particles of matter, a subject still of vital interest: I wish to talk about what may very loosely be termed the limits of measurement, a very closely related matter. I hope I have anticipated the wishes of my audience when I say that I shall not treat this subject too technically or mathematically, and that those of you who have met these issues in a professional capacity will pardon the use of simple and possibly inaccurate, although I hope not misleading, analogy. In particular, I make these apologies to our Chairman, Professor L. Rosenfeld, who with Neils Bohr has contributed a great deal to the exact formulation of the quantum mechanical limitations imposed on physical measurement.

This general issue—what are the limits to the range of possible physical measurements of different sorts?—has stayed in the forefront of interest in physics for a long time. Indeed, not only in physics but in many other sciences one frequently faces similar questions, and in some ways this is an all-pervading question embedded in the so-called " philosophy " of science. It is not, however, in quite such general terms that I intend to

המאמר נכתב ופותח על ידי פרופ. סמואל דבונס.
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MANCHESTER

discuss this problem to-day. I shall, rather, try to show how this problem has emerged to the forefront in the historical development of physical measurement, the extent to which there has been some recent clarification of the principles involved, and finally where the current interest lies as regards physical implications.

There are many directions in which we meet " practical " or " fundamental " limits when we try and extend the range of application of familiar physical measurements. We meet this problem of the limiting significance of measurement in answering any such questions as the following. What are the smallest particles, distances or structures that we can hope to " see "? What, if any, is the smallest interval of space (or of time, or space-time) that may have physical significance? What is the smallest electrical disturbance we can detect? (in a given time?). Is there a largest interval of space or time that we can measure, or that can have any physical significance? How does the smallest quantity of motion (or more exactly " action ", i.e. the product of energy and time) that can be effective in a physical process affect possible measurements of this or other quantities? Similarly, what relation is there to these limits and the impossibility (as we believe) of measuring our " absolute " motion, or observing any physical velocity which exceeds that of light?

Closely related to this question of the limits of measurement is the problem of the limiting precision with which a particular type of quantity can be measured. Essentially the same physical principles are involved in both cases. Likewise in both cases we meet with different results according to the circumstances: whether the measurement is an isolated one of the particular event or process, or one of a series involving some property which is a recurrent or repetitive feature; or possibly a characteristic which is believed to be constant and permanent and which may thus be subjected, in principle, to an indefinitely large number of similar measurements.

Essentially, then, I shall be discussing limits of a quantitative nature only, and only incidentally dealing with the significance or " reality " of using physical concepts at these limits. To-day, we believe we can recognise some types of limits as "fundamental",

whereas others are related to specific circumstances. The recognition of fundamental limits might suggest that there are some related ultimate boundaries to the scope of physical measurements or investigation, beyond which we cannot hope to penetrate. I myself do not believe that any of the limits we shall discuss represents an impenetrable barrier to further physical knowledge. As we extend our range of measurement the interpretation we place on the results may well shift, but as we adapt our attitude to new situations the barriers recede before us.

We might first consider the measurement of such simple quantities as mass, length or time in any unfamiliar range of magnitudes (and especially near the limits of what is practicable or possible) in relation to their measurement in the most familiar range. This latter range is, of course, that associated with the scale of the human body and its functions. We find, immediately, that in order to make measurements beyond the familiar range, we need some *extrapolation* of the normal concepts. However, this extrapolation is not necessarily a unique one. Any extrapolation involves a " theory," and in the present context a physical theory, and several such " theories " may be available as bases for extrapolations. If the extensions of our methods are valid and useful ones then we would like the different extrapolations to give results and conclusions which are consistent with one another.

For purposes of illustration I shall choose the measurement of the size and shape of physical objects—especially small ones. Vision is the most powerful of our senses and the one most intimately related to physical measurement, so that optical properties are particularly suited to our purpose. Let us consider the simple property of measuring the size of a simple object of " normal " dimensions: a coco-nut. Firstly, we have the direct method—application of rulers and similar measuring equipment. But suppose we have to deal with a large crate packed with a specified number of nuts. Then adopting a *theory*—that all the coco-nuts are of the same size and shape—and further, by estimating the closeness of packing, we can deduce the size of a single nut from the size of the crate. A quite different principle is illustrated by a coco-nut shy at a fairground. Armed with a sufficient number of balls, which we assume we aim randomly within certain limits, we can, again by adopting a *theory*, estimate the size of the coco-nuts. Our theory would be, say, that any collision between a ball and coco-nut leads to the dislocation of the latter, and our measurement would be of the chance of hitting the coco-nut with a given number of throws. Both these simple illustrations have close counterparts in physical methods actually used for measuring the sizes of atoms or molecules. At the other end of the scale, when we measure very large dimensions we again utilise the process of extrapolation involving some physical principle or theory: for example, in astronomy we utilise prior knowledge of the magnitude and constancy of the velocity of propagation of light. In other astronomical measurements of size, more specific physical theories are involved.

Apart from these dominant features—extrapolation and the use of a physical theory—there are other features of the measurement of size and shape which are typical of many types of physical measurement. Thus in some measurements we utilise some recurrent property and perform a process of averaging; in others the examination may be indirect and involve some physical law of inter-action; and in some cases the result is only obtained from a repetition, many times, of the process of measurement.

I shall turn now to the more specifically physical problems involved in measurement, and continue to use size and shape as the main illustration. We can distinguish three phases, historically, in the development of methods of extending the limits of physical measurement. These three phases, which very roughly we might call: *I*, the pre-Nineteenth century; *II*, the Nineteenth-century or classical outlook, and *III*, the post-Nineteenth century, involve different types of physical considerations.

I. The earliest limits to the range of possible measurement were imposed by the limited sensitivity of the detecting or measuring equipment involved. The most primitive physical measurements involve the senses directly, without any instrumental aids, so that any measurement with no direct physical sensation, sight, sound, touch, etc., is beyond the possible range. Extension of the range of measurement involves the use of more sensitive or amplifying equipment such as microscope (for distance), telescope (for angle) or electrical amplifier (for electric current or voltage). In the early development of physical measurements great emphasis was placed on this aspect and the limits of useful amplification were not always appreciated. However, to-day in the case of many, if not most, types of measurement instruments of sufficient magnifying or amplifying power are available, and with these we can transform the quantity being investigated into a quantity capable of direct appreciation by the senses. But, as we now know, increased magnification alone is of limited value since in any particular case such magnification eventually reveals either the limitations of the magnifying instrument, or some irreducible extraneous feature associated with the object or quantity being investigated.

I do not wish to suggest that in all physical measurements we do possess, at the present time, as much " magnification " as would be useful. For example, in studying small structures of atomic and molecular dimensions with the electron microscope there is still room for some practical improvement in the significant magnification. In measuring small time intervals between single events we still meet situations where the possession of more sensitive techniques would permit us to obtain more detailed, useful information. Again, in measuring signals from the distant parts of the universe we can detect weaker or more distant phenomena by increasing the size, and thereby the sensitivity, of our detector. However, in all these cases the rôle of amplification alone is clearly understood; and in any case we must also take into account both the intrinsic limitations of the instrument itself and the background disturbances against which the measurements are made. In principle the same considerations apply generally to all types of physical measurement, although traditionally rather different factors have been taken into account for different types of measurement. For example, the measurements of distance and of electric fields are related to the physical senses in rather different ways. Apart from the obvious, and limited, sensation of electric shock, electrical measurements are always transformed and measured via the other physical senses, usually vision. The concept and measurement of size is however, more directly related to the senses.

Corresponding to this difference, rather different criteria developed, historically, for the assessment of optical and electrical measurement equipment, and in particular for the maximum useful amplification.

II. The next phase in the extension of the range of physical measurements concerns the question of *resolution* rather than *magnification*. It is no longer a question of whether or not the physical quantities being measured can be magnified sufficiently to be appreciated by the senses, but rather one of how faithfully does the magnified " picture " correspond to the original, and to what extent are we magnifying the quantity which we wish to study, as opposed to some extraneous quantity. Both these aspects of the problem of resolution, or maximum useful magnification, were explored in the Nineteenth century on the basis of classical physics. On the basis of these classical theories one was led to the conclusion that although in any real situation such limits exist, and could be assessed quantitatively, in principle no limit exists, i.e. one could go on developing measuring techniques and " improving " the conditions of examination, thus continually and indefinitely extending the scope of significant measurement. Physics to-day, as we shall see, takes rather a different view of the situation.

Let us examine two types of limitation known to classical physics: (*a*) what we might term the " instrumental resolution ", and (*b*) the " background agitation ".

(*a*) In the microscopic examination of the structure of small objects a limit is set to the amount of detail that is revealed, even when ample magnification is available, by the wavelength of the light used to illuminate the objects. Roughly speaking, we cannot expect to *see* detail in the image which corresponds to structure of dimensions less than a wavelength, λ, in the object. Similarly, in the examination of distant objects with a telescope we cannot separate out clearly details which have an angular spacing, at the observing telescope, of less than λ/D radians, D representing the size of the telescope objective. With the value of λ for ordinary visible light, some 5×10^{-5} cm., these limits are quite familiar. Obviously, we cannot see atomic or molecular

dimensions (of the order 10^{-7}—10^{-8} cm.); and with a practical value of D of say 200 cm., the minimum angular resolution will be 10^{-5} of a degree or more, i.e. most stars will appear indistinguishable from point sources of light. However, in accordance with the spirit of classical physics there are no reasons, apart from practical ones, why we should not use as short a wavelength as we wish, and so improve the resolution by as large a factor as we choose. Of course, we may have difficulty in making optical lenses or mirrors for such short wavelengths (and in the astronomical case no radiation may penetrate the earth's atmosphere), but these are problems of technique rather than of principle.

(*b*) The second major problem which we meet with in studying small objects is that of keeping the object stationary whilst we examine it. This is a particular case of a general phenomena. Due to the finite temperature of our instruments and the neighbourhood of the objects under examination, all the particles of the system participate in a general thermal agitation whose magnitude increases with the absolute temperature. If we arrange our measurements over a long enough period (for example when we measure a weak electrical signal), these thermal fluctuations average out. Also in the measurement of the average position of some small particle, the thermal vibrations will not lead to any systematic error, but the vibrations do lead to a blurring out of the image and thus a loss in resolution. Since, according to classical physics, the extent of these movements is proportional to the absolute temperature, we have a simple remedy at hand: in principle, at absolute zero all the particles of matter come to rest, so that we have only to cool the substance under investigation sufficiently to reduce these thermal vibrations to a negligible amount! According to our present-day quantum mechanics all matter still comes to " rest " at the absolute zero, but this state of rest is not to be understood in the classical, Nineteenth-century, sense.

III. The quantum-mechanical interpretation of both the types of limitations we have discussed, differs substantially from the classical view. As regards the use of shorter and shorter wavelengths, this will be accompanied by definite interaction with the object being " viewed ". The simple, ideal, separation of object and instrument is no longer possible, even in principle. For, associated with every wavelength, λ, there is a definite finite momentum, p, which increases as the wavelength decreases:

$$p = \frac{h}{\lambda},$$

where h is the universal Planck's constant. Although h is very small, about 6×10^{-27} ergs-sec., its finite value is significant when we are dealing with very small objects and short wavelengths. As a consequence of this association of wavelength and momentum, any scattering of radiation, which is an essential in all " viewing " processes, leads to a transfer of momentum and energy to the object viewed; and this transfer increases as the wavelength is decreased, i.e. as we probe smaller and smaller distances. This relationship between the size or distance measured and the related, simultaneous, disturbance is a particular example of the famous " Uncertainty Principle ". We cannot conclude, from this principle, that it is impossible to measure distances or investigate structures smaller than a particular amount, but rather that if we do so we must be aware of the associated mechanical effects, and therefore we must be more careful in interpreting what it is that we do, apparently, measure.

In a closely related way, any attempt to " pin down " exactly the position of the object under investigation leads to complications of a quantum-mechanical nature. Even if our object is in surroundings at the absolute zero of temperature it is still not at rest in the classical sense. On the contrary, according to the uncertainty principle, if the position were well defined the momentum would be uncertain by a large amount. In practice the forces holding objects stationary on, say, a microscope slide are atomic forces of finite magnitude. The momenta of the atoms are finite and so also is the spread in their positions. (The product of these two qualities is again h.) In ordinary matter, then, we cannot expect to locate particles precisely to distances much smaller than atomic dimensions and we cannot, therefore, expect to " see " images of such particles in the normal sense which reveal detail much beyond atomic dimensions.

We might conclude from these considerations that we cannot hope to make a picture of the size or shape of any object much smaller than an atom. This is not the case however if we extend our concepts of seeing by quite a small step—a process of extrapolation of the sort already mentioned. Let us consider then the essential features in the process of seeing an object. Firstly, we must illuminate the object with light or other radiation and it is necessary for there to be some known (or estimable) type of interaction between the radiation and object. Each small part of the object acts as a centre of scattering of radiation, i.e. as a virtual " point source " of secondary radiation. The divergent bundles of rays from each of these secondary sources is then focused by a system of lenses so as to produce, in the ideal case, a " point " image in the focal plane of the instrument. (In the simplest case, direct vision, this plane is the retina of the eye.) Due to the finite wavelength of the radiation involved (and other, more practical, limitations of the optical system), the image of an ideal point object is not a perfect point, but is somewhat blurred. Nevertheless, if the overall picture has the same general spatial features as the object, we would regard this as seeing the object. Clearly, seeing objects in this sense is quite out of the question if the object cannot be held reasonably stationary.

If we extend our concept of viewing so as to include the general study of scattered radiation from an object, whether or not the image is a simple reproduction, even approximately, of the object, then we can also extend measurements to much smaller separations and objects. For example, if we illuminate our object with an extended parallel beam of radiation (plane-wave) and study the scattered radiation in various directions at large distances from the object, then motion of the object *as a whole* will not affect the observed distribution of the radiation. This distribution will depend only on the internal spatial relationships of the object, and not on the exact position of the object as a whole with respect to our measuring instrument. Needless to say, the resulting radiation pattern now is not what we would normally call a " picture " of the object; but if the wavelength is sufficiently small (of the same order as, or less than, the dimensions we are exploring), and the measurements

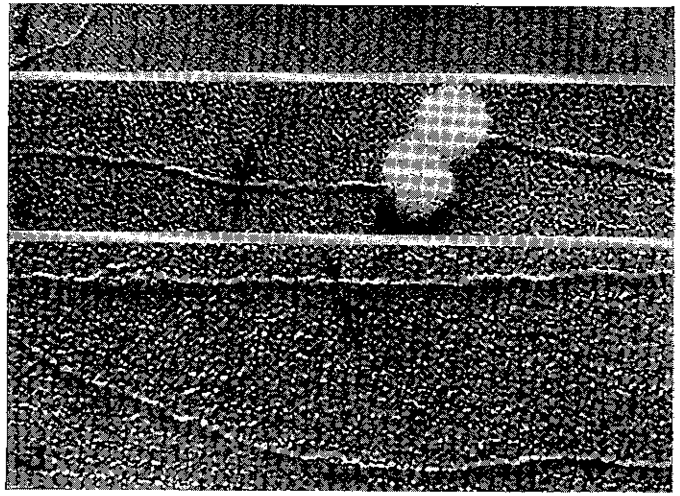

Fig. 2. *An electron-microscope photograph showing single molecules of DNA (deoxyribonucleiic acid) approximately* 2×10^{-7} *cms. in diameter (shadowed with platinum).*

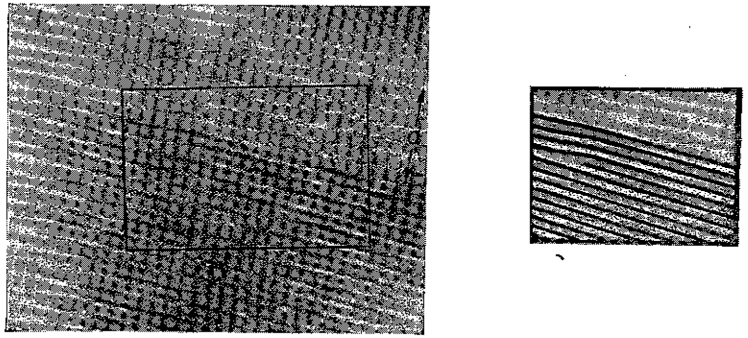

Fig. 3. *An electron-microscope picture showing single edge discolouration of platinum phthalocyanine crystal (magnification about* 500,000). *Diagram on the right is the schematic of the area marked.*

sufficiently exact and extensive, then we can by proper analysis, infer from the radiation pattern the real spatial structure of the object.*

One might say that if all this analysis and inference, which in practice involves extensive machine calculation, were done normally and instinctively by the human brain, then there would be very little difference between this general, and the more special types of "seeing", which I have described. However, even the most highly trained brains, with extensive experience of "seeing" in this general sense, are still a long way from such a state of skill!

To illustrate these rather general notions, let us look at pictures of objects ranging from the familiar size of everyday objects (the "coco-nut class"), to the smallest particles involved in nuclear physics—the constituents of the atomic nucleus, the so-called "nucleons".

When we view and describe an ordinary, macroscopic object, we are of course dealing only with an average overall picture. We use, in physics, the language of classical mechanics because this is of sufficient accuracy. From the viewpoint of quantum-mechanics the object we are viewing is a very complicated super-position of many more exact dynamical systems, but the difference between these is so small that the classical, *average*, picture is sufficiently precise for all practical purposes. Nevertheless, it is, in principle, only an *approximate* picture.

As we proceed to the study of smaller objects the situation is roughly unchanged (although our approximation is becoming progressively less exact), until we deal with objects of dimensions less than about 10^{-4} cm. In this region the finite-wavelength of the normal light used in vision becomes important. Nowadays this boundary is only of technical importance since equipment using electrons, with wavelengths many thousands of times shorter than light, is available: the electron microscope. Direct viewing, in the restricted sense, can be extended by a further factor of 1,000 to 10,000, down to molecular and atomic dimensions 10^{-7}—10^{-8} cm. In this region the problems we have

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* The two types of "seeing" discussed here are closely related to the two classes of optical diffraction called "Fresnel" and "Fraunhofer" respectively.

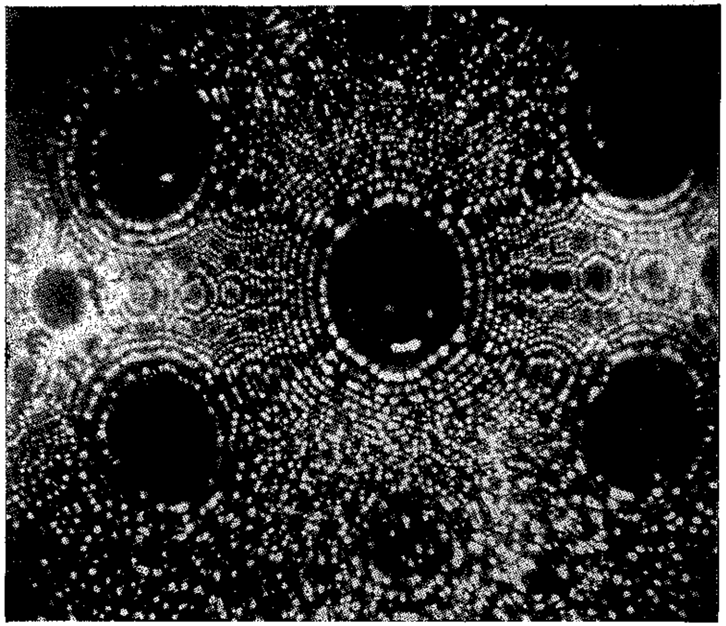

Fig. 5. Helium-ion image of tungsten tip of radius about 10^{-5} *cms. This picture shows a larger area of the tip than is displayed in Fig. 4. The improved resolution results from a reduction in temperature.*

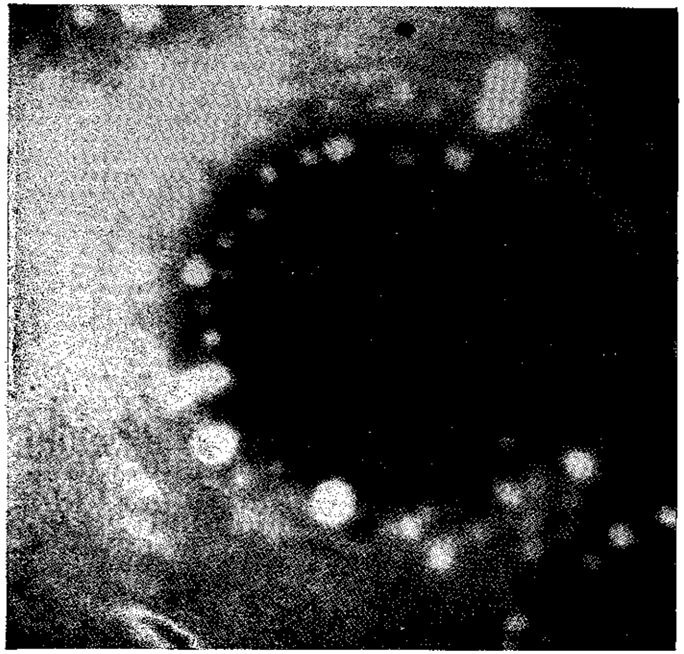

Fig. 4. Helium-ion image of a tungsten tip of radius 10^{-5} *cms., showing regions in the vicinity of the O11 plane with individual lattice steps of about* 5×10^{-8} *cms.*

discussed, such as blurring due to finite wavelength and motions of the object viewed, again become important even for electrons. In addition we meet problems in trying to assess how electrons interact with, i.e. how they are scattered by, the object under investigation. The range of sizes that can be investigated by direct viewing, i.e. by some sort of microscope, is illustrated in Fig. 1. Fig. 2 shows an electron-microscope photograph which

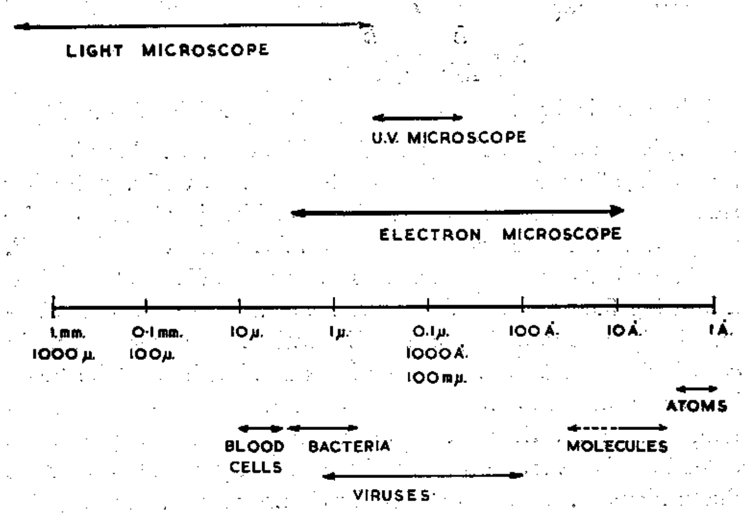

Fig. 1. *Range of sizes that can be seen with different sorts of microscopes.* ($1\mu = 10^{-4}$ cm.; 1Å $= 10^{-8}$ cm.)

reveals individual large molecules. Here we are still in the region where classical concepts are useful, although some extension of microscopic technique is involved: a special type of "staining" consisting of adding a very thin layer of heavy substance (platinum) to the object to produce visible shadows (so called "shadow casting").

Fig. 3 shows a regular array of large molecules with a localised dislocation. The correspondence between the image and the expected arrangement, on the right, indicates the extent to which we are dealing with a picture in the conventional sense.

Still smaller details can be "seen" in a particular type of instrument known as the "field emission microscope" in which the "object" is placed on the tip of a very fine needle point and

Fig. 6. *Optical synthesis of molecules of hexamethyl benzine.*

electrons, or other particles, leaving the point produce a magnified image.

Fig. 4 shows such a picture, involving helium ion emission, of the tip of tungsten " needle ". Single atomic planes are visible and even very small atomic clusters. The use of helium ions, in place of the more familiar electrons results in higher resolution. The wavelength (λ) associated with a particular particle of moderate energy is given by the De Broglie relationship

$$\lambda = \frac{h}{Mv} = \frac{h}{\sqrt{2ME}}$$

(v is the velocity and E the energy of the particle of mass M).

For a given energy, helium particles will have a wavelength some hundreds of times shorter than electrons. In Fig. 5 further resolution is obtained by reducing the blurring due to finite temperature. In this picture individual atoms can be " seen ".

The effects of temperature are not normally considered as a limiting factor in visual resolution because so often the wavelength is a more important limitation. Nevertheless, in considering the ultimate limits of observation it must be taken into account. In measurements other than microscopic ones the temperature limitations are often dominant, for example in the detection of weak radio signals. Until recently these temperature fluctuations have been considered primarily from the viewpoint of classical electrodynamics, but in the last couple of years it has become a matter of practical as well as theoretical importance to consider their essential quantum-mechanical aspects, particularly when we are concerned with limiting possibilities. This does not necessarily mean that the temperature fluctuations impose a more drastic limit when considered quantum-mechanically rather than classically: on the contrary, recent investigations have shown that " quantum-mechanical " amplifiers and detectors offer definite prospects of exploiting the improved threshold for detection which should, theoretically, result from working at extremely low temperatures near the absolute zero.

The illustrations so far have shown how we can progress from macroscopic to atomic dimensions without any very fundamental change. We have also seen the sort of fundamental problems that arise if we attempt to push further, and how these

can be circumvented. As an illustration of indirect viewing Fig. 6 shows a reconstructed " picture " of the molecule hexamethyl benzene. The essential information is obtained from diffraction patterns, i.e. measurements of the variation with angles of scattered radiation, in this case referred to as " optical synthesis ". In this picture we now have details of the internal structure of the molecule—a step beyond the pictures showing individual atoms or molecules as a whole. If we can use these techniques for atoms and molecules, what is to prevent us from applying similar methods to still smaller structures, say atomic nuclei, whose dimensions are only 10^{-4} or 10^{-5} those of atoms?

The first requirement will be a sufficiently short wavelength (10^{-13} cms. or less), but in addition we need to understand, as already mentioned, something about the way in which the radiation, or particles, we intend to use interacts with the object under investigation. The interaction of electrons with nuclei is fortunately sufficiently well understood for us to employ these particles as probes to investigate nuclear shapes and sizes. To obtain sufficiently small wavelengths very high energies, some hundreds of millions of electron-volts, are required, and as a result very large and elaborate equipment. We shall not expect a " picture " of the nucleus in the conventional sense, but by observing the way in which the electrons are scattered at different angles information about the structure of the nuclear particles can be inferred. This information could be portrayed by the blackening of a photographic plate, as in a normal photographic image, but it is more useful, and accurate, to show it mathematically.

In Fig. 7 measurements of the scattering of electrons by single protons are shown, and compared with various theoretical predictions, all of which are based on the assumption that a proton behaves like an ideal " point " particle, but with different types of interaction assumed. From these and other similar results it is possible to estimate the " size " of the proton as about 8×10^{-14} cm.* This represents just about the " smallest "

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* To obtain this result it is necessary to make the assumption that the electromagnetic interactions involved are similar to those which one meets in the case of much larger distances. Logically one can interpret the experimental measurements either in terms of a proton with a definite charge distribution of the magnitude stated, or in terms of some new, otherwise unknown, interaction with different " dimensions " (including a point particle).

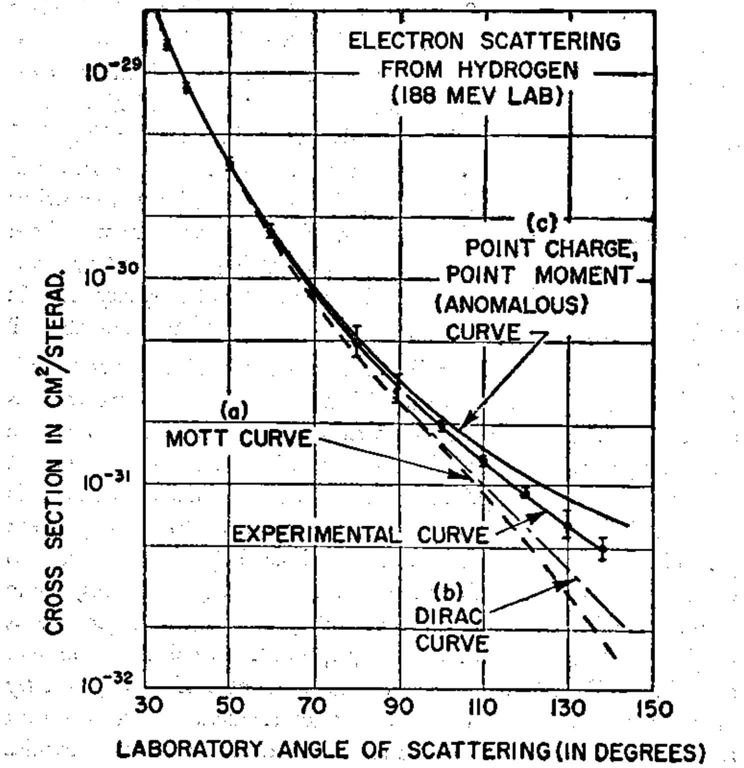

Fig. 7. The scattering of high energy electrons by protons as a function of angle.

object that has been investigated in physics up to the present. I have skipped over much of the intermediate ground between atoms and nucleons, in particular the extensive measurements that have been made of the nuclear sizes and shapes in the region 10^{-12}—10^{-13} cm. Much of the most precise information has been obtained by this sort of high energy electron-microscopy which has been used to investigate the proton structure, but many other less direct methods have also been used. The important feature of all these measurements is that the several different ones are all reasonably consistent with one another, a result which gives physicists confidence that they are talking sensibly when dealing with dimensions and structures as small as 10^{-13} cm.

It might now appear that there is a clear open road to the investigations of size indefinitely small by straightforward extension of the methods that have been described; and yet there is a feeling amongst many physicists that a serious qualitative change occurs somewhere around 10^{-13} cms. Why is this so? Some indications of the reason for this feeling can be found if we consider how we would make particles with wavelengths much smaller than 10^{-13} cm. The basic relationship

$$\lambda = \frac{h}{mv}$$

shows that if we use simple particles such as protons, we must eventually make mv much larger than m_0c where m_0 is the normal " rest mass " of the proton and c the velocity of light, i.e. the maximum velocity possible. This can only be done by giving the proton an energy large compared with its own intrinsic, or " rest mass ", energy. When particles of such large energy make close collisions with other similar particles, then, as we know now, new types of unstable particles (mesons and hyperons) may be produced, and these represent a much more complicated form of interaction than anything that is met with in the scattering of light or electrons by atoms or molecules. At least, these new processes are far less understood than the ordinary atomic and molecular processes.

There is, then, an essentially new feature involved in pushing our limits of measurement much beyond 10^{-13} cm. Up to this region we can, by choosing appropriate radiation or particles, find a means of exploration which does not involve energies comparable with the rest-masses of the particles involved, and which do not take us beyond the threshold where all the new types of particle are materialised. The problems therefore stay simple—or at least familiar. Beyond this sort of limit, the nature of particle-particle interaction seems to change dramatically. It may be possible to exploit some simpler interactions involving particular particles, just as electrons are used for probing nuclei; but such interactions must not only be simple, they must also be sufficient to reveal all that is significant in the structure being examined.

We can now see that with any particle, rest-mass m_0, that there is associated a fundamental length, $L \simeq h/m_0c$ (and also a time T equal to L/c). If we think of these as the smallest lengths to which we can give more or less unambiguous interpretation, then we can get some idea of the range of our physical measurements (or concepts) by comparing these limits with the values for the "size" and "age" of the Universe. The rough values are given in the table:

	L	T
Electron	10^{-11} cm.	10^{-21} sec.
Proton	10^{-14} cm.	10^{-24} sec.
Size and Age of Universe	10^{28} cm.	10^{18} sec.
(Ratio: $\frac{\text{Proton}}{\text{Universe}}$	10^{-42}	10^{-42})

The present range of magnitudes, $10^{42} : 1$, over which the concepts of length and time are used is enormous: nevertheless it is limited. So far no physical meaning is attached to lengths greater than 10^{28} cm. or less than 10^{-14} cm., and quantities outside these limits do not appear in physical theories or formulae. The final question I wish to put is: can we expect these limits to be final ones? Any attempt to predict the extent to which the limits will be extended can only be some dubious concoction of prophecy and numerology. Nevertheless if we glance at the way in which our ideas of measuring length developed historically, and how they are extended and modified to cover the range from the astronomical to the sub-atomic, I think we can get some idea of the general pattern of development.

I have emphasised that in extending any physical method of measuring extrapolation of ideas plays a major rôle. Successive extrapolations can lead in a perfectly systematic way to an overall change in going from one extreme to another. Moreover, the mode of extrapolation is not unique. I may, perhaps, illustrate this with a simple diagram:

$$\begin{array}{ccccccccccccccc}
 & & & & & & & a & b & c & d & & & & \\
 & & & a^1 & b & c & d & & b & c & d & e & & & \\
 & & a^1 & b^1 & c & d & & & & c & d & e & f & & \\
 & a^1 & b^1 & c^1 & d & & & & & & d & e & f & g & \\
a^1 & b^1 & c^1 & d^1 & & & & & & & & e & f & g & h
\end{array}$$

We start with some familiar concept or measurements having properties $a\ b\ c\ d$. A first extrapolation takes us to $b\ c\ d\ e$, the next to $c\ d\ e\ f$, etc., until at $e\ f\ g\ h$ we have a property *entirely* different from $a\ b\ c\ d$, but derived from it by a natural development. Similarly $a^1\ b\ c\ d$, etc., represents another, different, development. The different final properties may have little directly in common with each other, but they have this in common—that they were developed by a natural process of development from a common starting point. I believe many of our concepts and measurements in physics have this type of developing property. The successive steps in development and extrapolation are not all of the same magnitude or importance. As we extend our concepts, we must expect new features to arise, and although we may use the same terms, and some features of the earlier stage of development will persist, we must expect to find the significance of new measurements ever further removed from their original meanings. If we view the measurements of physical science in this way, including such basic ideas as length, size, shape, etc., then we can hardly conceive of any limits at all, other than those of man's inventive capacity—experimental and theoretical.