

# **Teachers' Hearing and interpretation of students' talk and action while they engage in mathematics problem solving**

Thesis for the degree Doctor of Philosophy

By Tali Wallach

Submitted to the Scientific Council of the

Weizmann institute of Science

Rehovot, Israel

February, 2005

## **Abstract**

Teachers are expected nowadays to assess student learning not only by a separate activity specifically designed for this purpose, but rather as an integral part of instruction. For example, by observing and listening to students solve mathematics problems in class. This study examines the structure and content of teachers' understanding and interpretation of students' talk and action while engaged in mathematical problem solving.

Participants in this study are 12 elementary school teachers who participated in an in-service workshop, led by the researcher. After solving several mathematics problems and discussing their solutions in small groups, each teacher was asked to choose one of the problems and to present it to a pair of students from her own class. The teachers observed and videotaped the students as they worked on solving the problems. Then, each teacher summarized and reflected on her observations in writing and met with the workshop leader to discuss episodes from the students' videotape.

Data collected include the workshop leader's journal, written work prepared by the teacher, the students' written work on the problem, and video-tapes of the following: all workshop sessions, the pairs of students' problem-solving sessions, individual interviews with each teacher that centered on episodes that the teacher chose from the videotape of her students, two focus-group interviews.

Data analysis is based on the Phenomenological Hermeneutics method and the "Grounded Theory" method. In line with the Phenomenological Hermeneutics method we distinguish between two forms of analysis: structural analysis and content analysis. Using the grounded theory method we coded the data from the interviews (of 12 teachers in the structural analysis and of 4 teachers in the content analysis) and generated initial categories, which were constantly compared with new data from the interview

and from the other sources. We refined them and identified core categories, and used them as a source for theoretical constructs.

Following are the main outcomes:

Structural analysis of the individual interview data centers on two dimensions: type of interpretation and focus of interpretation. The latter examined the focus in the teachers' interviews on the following three aspects in the students' work: cognitive, socio-cultural and affective. Data analysis indicates that all 12 teachers referred in their interview to all three aspects, but to different extents, focusing mainly on cognitive characteristics, and paying the least attention to affective characteristics. Averages of the relative frequencies of the aspects, which the teacher refers to, are: 70% - cognitive aspect (range 49% to 87%); 19% - socio-cultural aspect (range 5% to 44%); and 11% - affective aspect (range 3% to 20%).

Four types of teachers' interpretations of students' talk and actions were identified: (a) reporting – the teacher reconstructs what the students were saying/doing, (b) meaning – the teacher explains or justifies the students' talk or actions, (c) associating – the teacher connects the event to the students' educational/social/cultural history, and (d) inferring – the teacher connects the event to herself, to her role as a teacher, or to potential future actions of hers. The first two types – reporting and meaning – relate to the event itself, to the students' work, and are defined as internal interpretations, whereas the other two types reach out of the event to other contexts in order to understand and interpret it, and are defined as external interpretations. Data analysis indicates that the interviews of 11 teachers include all four types of interpretation, and one interview includes all but the inferring type. Again, the averages of the relative frequencies of the interpretations' types show different distributions, where the meaning type was most dominant on average and inferring and reporting the least. The averages of the relative frequencies of the interpretations' types are: 11% - reporting (range 1% to 20%); 63% - meaning (range 41% to 83%); 17% - associating (range 4% to 28%); and 9% - inferring (range 0% to 20%).

A two dimensional analysis of the teachers' interviews (types and focus of interpretations) reveals four structural profiles of interpretation, defined as follows: (1) internal-cognitive interpretation – focus on the event (the students' work) with little connections to external contexts, together with an emphasis on cognitive aspects of the students' work, (2) internal-varied – focus on the event with reference to different aspects of the students' work, (3) external-cognitive – using external contexts to explain the event or inferring new insights, together with an emphasis on cognitive aspects of the students' work, and (4) external-varied – using external contexts with reference to the different aspects.

Content analysis of the data suggests two categories of characteristics related to teacher hearing (i.e., understanding students' talk and action). One category is defined as compatibility of teacher hearing with what the students are saying/doing. In this category the following characteristics were identified: over-hearing – the teacher hears things that were not said by the students; under-hearing – the teacher does not hear some of the things said or done by the students; no-hearing – the teacher does not hear a whole component of the students' work (the two last characteristics of hearing are similar but differ in magnitude), biased-hearing – the teacher hearing and interpretation are not in accordance with the students talk and action, compatible-hearing – full compatibility between teacher's hearing and students' talk and actions. All these characteristics, except the last one, describe different variations of partial compatibility.

The second category is defined as the complexity of teacher hearing. In this category the following two characteristics were identified: flexible-hearing – the teacher examines different alternatives and varied issues for explaining students' talk and actions; complex-hearing – the teacher hears components, which cannot be observed or understood directly from students' talk or behavior, but rather, entails analysis, generalization, and conceptualization of what students say/do.

This study offers insight into what it might mean for teachers to use the advocated new ways of student assessment. It provides a useful theoretical tool for conducting further research in this area, and important information for the development of pre-service and in-service programs to prepare teachers meet the challenges embedded in adopting the new ways of assessment.